15.4 Cumulative Distribution Function for Poisson Probability Distribution

A. Purpose

The procedure described in this chapter computes the Cumulative Distribution Function (CDF) of the Poisson probability distribution. The CDF is sometimes called the lower tail. The lower tail, or CDF, $Q(n|\lambda)$, and the upper tail, $P(n|\lambda)$ for the Poisson probability distribution with parameter λ and argument n are defined by

$$\Pr\{x < n|\lambda\} = Q(n|\lambda) = e^{-\lambda} \sum_{j=0}^{n-1} \frac{\lambda^j}{j!},$$

$$\Pr\{x \geq n|\lambda\} = P(n|\lambda) = e^{-\lambda} \sum_{j=n}^{\infty} \frac{\lambda^j}{j!} = 1 - Q(n|\lambda).$$

B. Usage

B.1 Program Prototype, Single Precision

REAL LAMDA, P, Q

INTEGER N, IERR

Assign values n to N and λ to LAMDA, and obtain P = $P(n|\lambda)$ and Q = $Q(n|\lambda)$ by using

CALL SCDPOI (N, LAMDA, P, Q, IERR)

B.2 Argument Definitions

N [in] Argument n of the functions $P(n|\lambda)$ and $Q(n|\lambda)$. N must be nonnegative. Must be positive if LAMBDA = 0.

LAMDA [in] Parameter λ of the functions $P(n|\lambda)$ and $Q(n|\lambda)$. LAMBDA must be nonnegative. Must be positive if N = 0.

P [out] The value of the function $P(n|\lambda)$.

Q [out] The value of the function $Q(n|\lambda)$.

IERR [out] A flag that normally is zero to indicate successful computation. See Section E below for discussion of non-zero values.

B.3 Modifications for Double Precision

For double precision computation, change the REAL type statement to DOUBLE PRECISION and change the initial letter of the procedure name to D.

C. Example and Remarks

See DRDCDPOI and ODDCDPOI for an example of the usage of this subprogram.

The procedures SGAMIK and SGAMIE, described in Chapter 2.19, are used to control the procedure SGAMI and determine the error estimate it returns. SGAMI is used as described in Section D below.

D. Functional Description

D.1 Method

The identities $P(n|\lambda) = P(a, x)$ and $Q(n|\lambda) = Q(a, x)$, with $a = n$ and $x = \lambda$, where $P(a, x)$ and $Q(a, x)$ are incomplete gamma function ratios, are used. The procedure SGAMI described in Chapter 2.19 is used to evaluate $P(a, x)$ and $Q(a, x)$.

D.2 Accuracy Tests

See Section 2.19.D.

E. Error Procedures and Restrictions

The procedure SGAMI issues error messages, under several conditions, at level 2 + MSGOFF, where MSGOFF is zero unless specified by a call to SGAMIK (see Chapter 2.19) at some time before calling SCDPOI. If error termination is suppressed by setting MSGOFF < 0, or by calling ERMSET (see Chapter 19.2), IERR will be set to a non-zero value.

If the desired tolerance could not be achieved, IERR is set to 2.

If SCDPOI is called with both LAMDA and N zero, IERR is set to 3 and P is set to 3.0.

If SCDPOI is called with one or both of LAMDA and N negative, IERR is set to 4 and P is set to 4.0.

F. Supporting Information

Entry	Required Files
DCDPOI	AMACH, DCDPOI, DCSEVL, DERF, DERM1, DERV1, DGAM1, DGAMMA, DINITS, DRCOMP, DREXP, DRLOG, DXPARG, ERFIN, ERMSG, IERM1, IERV1
SCDPOI	AMACH, ERFIN, ERMSG, IERM1, IERV1, SCDPOI, SCSEVL, SERF, SERM1, SERV1, SGAM1, SGAMMA, SINITS, SRCOMP, SREXP, SRLOG, SXPARG

Designed and programmed by W. V. Snyder, JPL, 1993.

DRDCDPOI

```
      program DRDPOI
c>> 2001-05-25 DRDCDPOI Krogh Minor change for making .f90 version.
c>> 1996-05-28 DRDCDPOI Krogh  Changed Fortran 90 code.
c>> 1994-10-19 DRDCDPOI Krogh  Changes to use M77CON
c>> 1994-07-06 DRDCDPOI WVS set up for chgtyp
c
c     Evaluate the Probability Function Q(n,lamda) of the Poisson
c     distribution by using DCDPOI.
c
c--D replaces "?": DR?POI, DR?CDPOI, ?cdpoi, ?gamik
      double precision LAMDAS(4), P, Q(4)
      integer J, N, IERR
      data LAMDAS /0.5d0, 1.0d0, 1.5d0, 2.0d0/
c
10    format ('      Poisson Probability Function Q(lamda | n)'/
     1        '  N    LAMDA =',4f14.7/)
30    format (i3,15x,1p,4g14.7)
      call dgamik (0.0d0, 0.0d0, 0.0d0, 0)
      print 10, lamdas
      do 50 n = 1, 10
         do 40  j = 1, 4
            call dcdpoi (n, lamdas(j), p, q(j), ierr)
40       continue
         print 30, n, q
50    continue
      stop
      end
```

ODDCDPOI

```
     Poisson Probability Function Q(lamda | n)
  N    LAMDA =      0.5000000      1.0000000      1.5000000      2.0000000

  1                 0.6065307      0.3678794      0.2231302      0.1353353
  2                 0.9097960      0.7357589      0.5578254      0.4060058
  3                 0.9856123      0.9196986      0.8088468      0.6766764
  4                 0.9982484      0.9810118      0.9343575      0.8571235
  5                 0.9998279      0.9963402      0.9814241      0.9473470
  6                 0.9999858      0.9994058      0.9955440      0.9834364
  7                 0.9999990      0.9999168      0.9990740      0.9954662
  8                 0.9999999      0.9999898      0.9998304      0.9989033
  9                  1.000000      0.9999989      0.9999723      0.9997626
 10                  1.000000      0.9999999      0.9999959      0.9999535
```